Puteri si radicali.

1. Prin **puterea n** a unui numar real **a** intelegem numarul $\mathbf{a^n = \underbrace{a \cdot a \cdot \ldots \cdot a}_{n}}$ $(\mathbf{n \in N})$

a se numeste **baza**, iar $\mathbf{n \in N}$ se numeste **exponent**.

Daca $\mathbf{a \neq 0}$ avem $\mathbf{a^0 = 1,\ a^{-n} = \frac{1}{a^n}}$.

2. Prin **radacina de ordin n** sau **radical de ordin n, $n \in N$, $n \geq 2$** a unui numar **a > 0** intelegem un numar real, pe care il notam cu $\mathbf{\sqrt[n]{a} = a^{\frac{1}{n}}}$ si care are proprietatea $\mathbf{(\sqrt[n]{a})^n = a}$.

<u>Proprietati - puteri:</u>
Fie $\mathbf{n, m \in N,\ a, b \in R^*}$

a) $\mathbf{a^n a^m = a^{n+m}}$, b) $\mathbf{(a^n)^m = a^{nm}}$, c) $\mathbf{\frac{a^n}{a^m} = a^{n-m}}$, d) $\mathbf{(ab)^n = a^n \cdot b^n}$,

e) $\mathbf{\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}}$.

<u>Proprietati - radicali:</u>
Fie $\mathbf{a, b > 0,\ n, m \in N,\ n, m \geq 2}$,

a) $\mathbf{\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}}$, b) $\mathbf{\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}}$, c) $\mathbf{\sqrt[n]{a^{n \cdot m}} = a^m}$, d) $\mathbf{(\sqrt[n]{a})^m = \sqrt[n]{a^m}}$,

e) $\mathbf{\sqrt[n]{a^m} = \sqrt[nk]{a^{mk}}}$, f) $\mathbf{\sqrt[n]{\sqrt[m]{a}} = \sqrt[nm]{a}}$.

3. Daca **a<0, $n \geq 3$, $n \in N$ impar,** se numeste **radical de ordinul n al lui a,** numarul negativ notat $\mathbf{\sqrt[n]{a}}$ care are proprietatea ca $\mathbf{(\sqrt[n]{a})^n = a}$.

Obs.: Proprietatile date in cazul radicalilor din numere pozitive sunt valabile si pentru radicalii de ordin impar din numere negative.

4. $\mathbf{\sqrt{A + \sqrt{B}}}$ si $\mathbf{\sqrt{A - \sqrt{B}}}$ se numesc **radicali dubli.** In anumite conditii acestia se descompun in **suma** sau **diferenta de radicali simpli.**

Daca $\mathbf{A^2 - B = C^2}$ (este un patrat perfect) atunci:

$$\mathbf{\sqrt{A + \sqrt{B}} = \sqrt{\frac{A + C}{2}} + \sqrt{\frac{A - C}{2}}} \quad \text{si} \quad \mathbf{\sqrt{A - \sqrt{B}} = \sqrt{\frac{A + C}{2}} - \sqrt{\frac{A - C}{2}}}$$

5. O expresie care contine radicali se numeste **conjugata** unei alte expresii care contine radicali, daca produsul celor doua expresii se poate scrie fara radicali.
Cele doua expresii se numesc **conjugate.**

Exemple:

a) $a > 0$, $b \in R$ atunci $\sqrt{a}+b$ si $\sqrt{a}-b$ sunt conjugate deoarece $(\sqrt{a}+b)(\sqrt{a}-b) = a - b^2$,

b) $a, b > 0$ atunci $\sqrt{a}+\sqrt{b}$ si $\sqrt{a}-\sqrt{b}$ sunt conjugate deoarece $(\sqrt{a}+\sqrt{b})(\sqrt{a}-\sqrt{b}) = a - b$.

6. Puteri cu **exponent rational**:

a) Puteri cu **exponent rational pozitiv**: definim $\mathbf{a^{\frac{m}{n}} = \sqrt[n]{a^m}}$, $\mathbf{a \geq 0}$ si $\mathbf{\frac{m}{n} \in Q}$, $\mathbf{\frac{m}{n} > 0}$, $\mathbf{n \geq 2}$,

b) Puteri cu **exponent rational negativ**: definim $\mathbf{a^{-\frac{m}{n}} = \frac{1}{a^{\frac{m}{n}}} = \frac{1}{\sqrt[n]{a^m}}}$, $\mathbf{a > 0}$ si $\mathbf{\frac{m}{n} \in Q}$, $\mathbf{\frac{m}{n} > 0}$, $\mathbf{n \geq 2}$

<u>Proprietati ale puterilor cu exponent rational</u>

Daca **a** > **0, b** > **0** si $\mathbf{\frac{m}{n}, \frac{p}{q} \in Q}$ avem:

a) $\mathbf{a^{\frac{m}{n}} \cdot a^{\frac{p}{q}} = a^{\frac{m}{n}+\frac{p}{q}}}$, b) $\mathbf{(ab)^{\frac{m}{n}} = a^{\frac{m}{n}} \cdot b^{\frac{m}{n}}}$, c) $\mathbf{\left(\frac{a}{b}\right)^{\frac{m}{n}} = \frac{a^{\frac{m}{n}}}{b^{\frac{m}{n}}}}$, d) $\mathbf{\left(a^{\frac{m}{n}}\right)^{\frac{p}{q}} = a^{\frac{m}{n}\cdot\frac{p}{q}}}$,

e) $\mathbf{\frac{a^{\frac{m}{n}}}{a^{\frac{p}{q}}} = a^{\frac{m}{n}-\frac{p}{q}}}$.

<u>Alte proprietati</u>

a) Daca $\mathbf{0 < a < 1}$ si $\mathbf{n \geq 2, n \in N}$ atunci $\mathbf{0 < \sqrt[n]{a} < 1 \Leftrightarrow 0 < a^{\frac{1}{n}} < 1}$.

b) Daca $\mathbf{a > 1}$ si $\mathbf{n \geq 2, n \in N}$ atunci $\mathbf{1 < \sqrt[n]{a} \Leftrightarrow 1 < a^{\frac{1}{n}}}$.

Pornind de la aceste proprietati putem stabili urmatoarele:

a) Daca $\mathbf{0 < a < 1}$ si $\mathbf{x \in Q, x > 0}$ atunci $\mathbf{0 < a^x < 1}$.
b) Daca $\mathbf{a > 1}$ si $\mathbf{x \in Q, x > 0}$ atunci $\mathbf{a^x > 1}$.
c) Daca $\mathbf{0 < a < 1}$ si $\mathbf{x \in Q, x < 0}$ atunci $\mathbf{a^x > 1}$.
d) Daca $\mathbf{a > 1}$ si $\mathbf{x \in Q, x < 0}$ atunci $\mathbf{0 < a^x < 1}$.
e) $\mathbf{(\forall x) \in Q}$ avem $\mathbf{1^x = 1}$.